When (B5) is equated to (11) and coefficients of $U_m(v)$ are matched, the following equation results:

$$C_{n+1} = \frac{-2}{\pi} \sum_{m=n}^{\infty} A_m R_m q_{nm}, \qquad n=0,1,2,\cdots. \quad \text{(B7)}$$

R_m in (B7) can be evaluated by transforming (B6) into an equation in terms of u, substituting for $F(u)$ from (13), and using the expansion given in (B3) and the orthogonality relationship given in (B4). The resulting expression is given as

$$R_m = \frac{\pi\beta}{2}(1-\delta_{m0}) \sum_{k=0}^{m-1} \frac{C_k}{\Delta_k} p_{k,m-1}.$$

Thus (B7) reduces to

$$C_{n+1} = -\beta \sum_{m=n}^{\infty} \sum_{k=0}^{m-1} (1-\delta_{m0}) A_m q_{nm} \frac{C_k}{\Delta_k} p_{k,m-1}, \qquad n=0,1,2,\cdots.$$

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The Compensation of Step Discontinuities in TEM-Mode Transmission Lines

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***Abstract*—A method for the compensation of the effects due to the discontinuities that arise when transmission lines of different characteristic impedance are joined is presented. The proposed method is not based on calculating the equivalent circuit of the discontinuity but makes use of a simple taper on the wider line at an impedance step to remove the effects of the discontinuity.**

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I. Introduction

The effects observed when lines of different characteristic impedance are joined to form a step discontinuity are well known, and various authors have presented equivalent circuits [1], [2]. The parameters of such equivalent circuits have to be incorporated into the design, which can lead to considerable complication. In the limiting case, the end effect observed in open-circuit stubs or open-circuit parallel-coupled lines can be regarded as a step from finite to zero linewidth. In the latter case,

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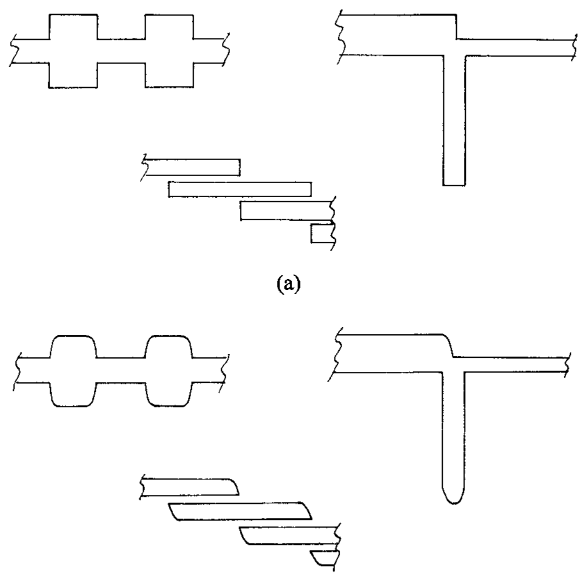

Fig. 1. Examples of (a) uncompensated and (b) compensated discontinuities.

compensation of the step is simple, as the discontinuity represents a lumped capacitance and the line can easily be shortened to compensate for this excess capacitance [3], [4].

In this paper, a method is proposed whereby any step discontinuity is compensated for by a gradual change in the conductor cross section in such a way that the characteristic impedances of the connected lines remain constant right up to the junction, and the junction capacitance vanishes. Fig. 1 shows three examples where such steps occur: 1) in those cases for a low/high-impedance filter, 2) the case where lines of differing characteristic impedance meet at the point where a stub is connected, and 3) for a side-coupled filter. The compensated networks are shown in Fig. 1(b).

II. Derivation

Consider a section of a transmission line which has been tapered to compensate for the discontinuity, as shown in Fig. 2. The chord AB (approximated in the limit by a straight line) is longer than CD. Consequently, the fringing capacitance over AB must be larger than that over CD. The basis of the approximation that follows is to exchange the self-capacitance of the line for the increased fringe capacitance in such a way that the total capacitance over the cross section remains constant with varying values of x, under the assumption that the propagation remains in the TEM mode. Higher order modes are neglected.

The total normalized capacitance of the element of the line $A'B'E'F'$, when occurring in an uncompensated section of the line, is simply the sum of its two (upper and lower) self-capacitances, and the fringe-field capacitance of $A'-B'$:

$$4\frac{w}{b}\,\delta x + 2C_f'/\epsilon\,\delta x \tag{1}$$

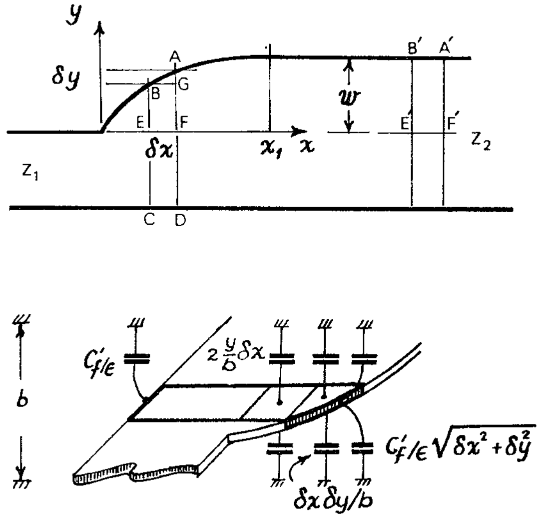

Fig. 2. Compensation for discontinuity.

where b is the ground-plate separation and C_f'/ϵ the normalized fringe-field capacitance [5].

In the compensated part of the line, the corresponding capacitance is given by the sum of the capacitances of $BEFG$, the triangle ABG, and the fringe-field of AB:

$$4\frac{y}{b}\,\delta x + \frac{4}{b}\left(\tfrac{1}{2}\delta x\,\delta y\right) + 2C_f'/\epsilon\sqrt{\delta x^2 + \delta y^2}\,. \tag{2}$$

The characteristic impedance of the line Z_2 will consequently remain constant if the two expressions (1) and (2) are equal over all x:

$$4\frac{w}{b}\,\delta x + 2C_f'/\epsilon\,\delta x = 4\frac{y}{b}\,\delta x + \frac{4}{b}\left(\tfrac{1}{2}\delta x\,\delta y\right) + 2C_f'/\epsilon\sqrt{\delta x^2 + \delta y^2} \tag{3}$$

therefore,

$$\frac{4}{b}(w-y) + 2C_f'/\epsilon = \frac{2}{b}\,\delta y + 2C_f'/\epsilon\sqrt{1+\left(\frac{\delta y}{\delta x}\right)^2}\,. \tag{4}$$

For convenience, let $C_f'/\epsilon = c$. Letting $\delta x \to 0$, $\delta y \to 0$, the term $2/b\delta y \to 0$, so that, in the limit,

$$\frac{4}{b}(w-y) + 2c = 2c\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,. \tag{5}$$

Rearranging,

$$\frac{dy}{dx} = \sqrt{\left[1+\frac{2}{bc}(w-y)\right]^2 - 1}\,. \tag{6}$$

Let

$$1+\frac{2}{bc}(w-y) = z \qquad \frac{dy}{dx} = -\frac{bc}{2}\frac{dz}{dx}. \tag{7}$$

Then

$$\frac{dz}{dx} = -\frac{2}{bc}\sqrt{z^2-1}\,. \tag{8}$$

Separating the variables and integrating yields

$$\ln\left[z+\sqrt{z^2-1}\,\right] = -\frac{2}{bc}x + K$$
$$= \cosh^{-1} z \tag{9}$$

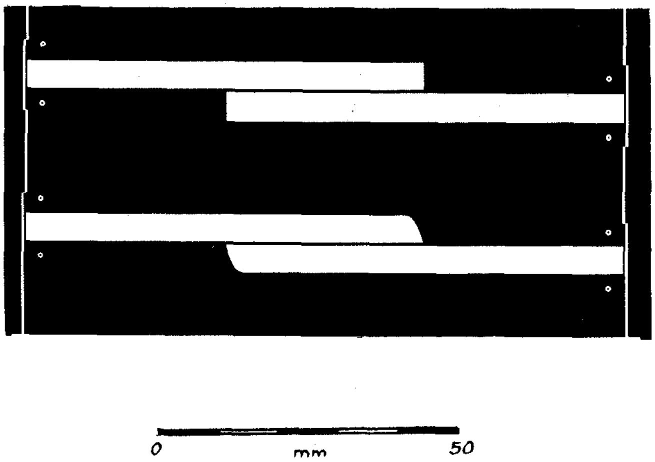

Fig. 3. Etching mask for a trial circuit.

where K is a constant of integration. Resubstituting (7) gives

$$z=\cosh\left[K-\frac{2}{bc}x\right]=1+\frac{2}{bc}(w-y) \tag{10}$$

therefore,

$$y=w-\frac{bc}{2}\left\{\cosh\left[K-\frac{2}{bc}x\right]-1\right\}. \tag{11}$$

The constant is eliminated by applying the boundary condition that if $x=0$, then $y=0$, yielding

$$K=\cosh^{-1}\left[\frac{2w}{bc}+1\right] \tag{12}$$

or

$$y=w-\frac{bc}{2}\left\{\cosh\left[\cosh^{-1}\left(\frac{2w}{bc}+1\right)-\frac{2x}{bc}\right]-1\right\}. \tag{13}$$

The distance away from the junction at which the compensation has been completed x_1 is found by setting $y=w$ and solving for x.

$$x_1=\frac{cb}{2}\cosh^{-1}\left(\frac{2w}{cb}+1\right). \tag{14}$$

III. Measurement

In order to evaluate the correctness of the proposed method of compensation, two circuits were prepared on 3.175-mm GPS Polyolefin stripline, as shown in Fig. 3, with one pair of lines being compensated and the other not. For the uncompensated lines, a minimum value of S_{21} can be expected at a frequency lower than the theoretical $\lambda/2$ resonant frequency, due to the stray capacitance at the open-circuit line ends. In the case of the compensated lines, these two frequencies should of course coincide.

TABLE I

Parameter	Uncompensated	Compensated
Theor. resonance frequency, GHz	3.011	3.029
True resonance frequency, GHz	2.971	3.024
Error, MHz	40	5
Error, %	1.24	0.17

The physical lengths of the circuits were accurately measured, and the corresponding theoretical resonant frequencies calculated. The true resonant frequencies were then obtained by noting the $\lambda/2$ resonance minimum in S_{21} by means of a network analyzer and frequency counter. These results are shown in Table I.

IV. Conclusions

The above measurements show a very close correspondence between the theoretical and practical values of resonant frequency for the compensated case, indicating that the discontinuity capacitance has been correctly compensated for. The method is applicable to steps of any size, including their end effect in open-circuit parallel-coupled lines; by using the necessary capacitance equations, the method can be extended to cover microstrip applications. The advantages of the method lie mainly in the fact that the compensation of discontinuity can be done at a manufacturing stage rather than in the design stage, giving a much less complicated design.

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